

# GAUGE MODEL IN D=3, N=5 HARMONIC SUPERSPACE

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## Abstract

We construct the Grassmann-analytic gauge superfields in  $D=3, N=5$  harmonic superspace using the  $SO(5)/U(1) \times U(1)$  harmonics. These gauge  $N=5$  superfields contain an infinite number of bosonic and fermionic fields arising from decompositions in harmonics and Grassmann coordinates. The bosonic sector of this supermultiplet includes the gauge field  $A_m$ , the additional nongauge vector field  $B_m$ , the scalar field  $S$ , two  $SO(5)$ -vector scalar fields and an infinite number of auxiliary fields with  $SO(5)$  indices. The nonabelian Chern-Simons-type action in the  $N=5$  analytic harmonic superspace is constructed. This action is also invariant with respect to the sixth supersymmetry realized on the  $N=5$  gauge superfields. The component Lagrangian describes the scale-invariant nontrivial interactions of the gauge Chern-Simons field  $A_m$  with  $B_m, S$  and other basic and auxiliary fields. All auxiliary fields can be excluded from this Lagrangian.

## 1 Introduction

Supersymmetric extensions of the three-dimensional Chern-Simons (CS) theory were discussed in refs. [1]-[12]. The  $N=1$  CS theory of the spinor gauge superfield [1, 2] was constructed in the  $D=3, N=1$  superspace with real coordinates  $x^m, \theta^\alpha$ , where  $m = 0, 1, 2$  is the 3D vector index and  $\alpha = 1, 2$  is the  $SL(2, R)$  spinor index. The  $N=1$  CS action can be interpreted as the superspace integral of the Chern-Simons superform  $\text{Tr}(dA + \frac{2}{3}A^3)$  in the framework of our theory of superfield integral forms [3]-[5].

The abelian  $N=2$  CS action was first constructed in the  $D=3, N=1$  superspace [1]. The nonabelian  $N=2$  CS action was considered in the  $D=3, N=2$  superspace in terms of the Hermitian superfield  $V(x^m, \theta^\alpha, \bar{\theta}^\alpha)$  (prepotential) [3, 10, 11], where  $\theta^\alpha$  and  $\bar{\theta}^\alpha$  are the complex conjugated  $N=2$  spinor coordinates. The corresponding component Lagrangian includes the bosonic CS term and the bilinear terms with fermionic and scalar fields without derivatives. The unusual dualized form of the  $N=2$  CS Lagrangian contains the second vector field instead of the scalar field [11].

The  $D=3, N=3$  CS theory was first analyzed by the harmonic-superspace method [6, 7]. Note that the off-shell  $N=3$  and  $N=4$  vector supermultiplets are identical [14], however, the superfield CS action is invariant with respect to the  $N=3$  supersymmetry only. Nevertheless, the  $N=3$  CS equations of motion are covariant under the 4th supersymmetry. The field-component form of the  $N=3$  CS Lagrangian was studied in [8, 9].

The physical fields of the  $D=3, N=8$  vector multiplet and the corresponding SYM Lagrangian can be found by a dimensional reduction of the  $D=4, N=4$  SYM theory. The algebra of supersymmetric transformations closes on the  $SYM_3^8$  equations of motion only,

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so it is not clear how to use these fields in a hypothetical supersymmetric generalization of the CS theory. The off-shell  $D=3, N=6$  SYM theory arises by a dimensional reduction of the  $\text{SYM}_4^3$  theory in the  $\text{SU}(3)/\text{U}(1)\times\text{U}(1)$  harmonic superspace [13]. The off-shell  $N=6$  gauge superfields contain the physical fields of the  $\text{SYM}_3^8$  theory and an infinite number of auxiliary fields with the  $\text{SU}(3)$  indices. The integration measure of the corresponding  $D=3, N=6$  analytic superspace has dimension 1, and we do not know how to construct the CS theory in this superspace.

In this paper, we consider the simple  $D=3, N=5$  superspace which cannot be obtained by a dimensional reduction of the even coordinate from any 4D superspace. The corresponding harmonic superspace (HSS) using the  $\text{SO}(5)/\text{U}(1)\times\text{U}(1)$  harmonics is discussed in Section 2. The Grassmann-analytic  $D=3, N=5$  superfields depend on 6 spinor coordinates, so the analytic-superspace integral measure is scale-invariant. It is not difficult to prove that this measure is also invariant with respect to the  $D=3, N=5$  superconformal group.

In Section 3, we introduce three basic gauge superfields in the  $D=3, N=5$  harmonic analytic superspace. The superfield formalism of this model is nominally similar to the HSS formalism of the  $D=4, N=3$  SYM theory [13], although Grassmann dimensions of two superfield models are different. One can construct the Chern-Simons-type (CST) superfield action from our three  $D=3, N=5$  gauge superfields which looks as a formal analog of the  $D=4, N=3$  HSS action. Note that one gauge superfield can be composed from two mutually conjugated gauge prepotentials. Our superfield CST action is invariant with respect to the sixth supersymmetry transformation realized on the  $N=5$  superfields.

The field-component structure of our  $D=3, N=6$  CST model is analyzed in Section 4. In the abelian case, the basic complex gauge superfield includes the gauge field  $A_m$ , the nongauge vector field  $B_m$ , the scalar field  $S$ , and the fermionic fields  $\mu_\alpha$  and  $\psi_\alpha$ . All other fields of the infinite off-shell multiplet carry  $\text{SO}(5)$  indices, for instance, the basic real fields  $v^a, \nu_\alpha^a$  and  $S^a$ . The component abelian Lagrangian contains Chern-Simons terms for  $A_m$  and  $B_m$ , the simple interaction  $S\partial_m B^m$ , and the bilinear interactions of other fermionic and bosonic fields. All abelian auxiliary fields with more than two  $\text{SO}(5)$  indices vanish on-shell, so one can construct the CST Lagrangian on the finite-dimensional  $N=6$  supermultiplet from our superfield action. The basic abelian fields  $v^a, \mu^\alpha, \nu_\alpha^a, S$  and  $S^a$  satisfy the free massless equations, the abelian solution for  $A_m$  is pure gauge, but the solution for  $B_m$  is nontrivial. The  $\text{SO}(5)$ -vector abelian auxiliary fields can be composed on-shell from the derivatives of the free fields  $v^a, \nu_\alpha^a$  and  $S^a$ .

In the nonabelian version of our CST model, the gauge field  $A_m$  interacts with other fields of the  $N=6$  gauge supermultiplet, so we cannot obtain the pure gauge solution for  $A_m$  in this case.

## 2 $D=3, N=5$ superspace

Let us consider the  $D=3, N=5$  superspace coordinates  $x^m, \theta_a^\alpha$ , where  $m = 0, 1, 2$  is the  $\text{SO}(2,1)$  vector index,  $a = 1, 2, \dots, 5$  is the vector index of the automorphism group  $\text{SO}(5)$ , and  $\alpha = 1, 2$  is the spinor index of the  $\text{SL}(2, \mathbb{R})$  group. We use the real traceless or symmetric representations of the 3D  $g_m$  matrices

$$(\gamma_m)^{\alpha\beta} = \varepsilon^{\alpha\rho}(\gamma_m)_\rho^\beta = (\gamma_m)^{\beta\alpha}, \quad (\gamma_m\gamma_n)_\alpha^\beta = -\eta_{mn}\delta_\alpha^\beta + \varepsilon_{mnp}(\gamma^p)_\alpha^\beta, \quad (2.1)$$

where  $\eta_{mn} = \text{diag}(1, -1, -1)$  is the 3D Minkowski metric. One can consider the bispinor representation of the 3D coordinates and derivatives:  $x^{\alpha\beta} = (\gamma_m)^{\alpha\beta} x^m$ ,  $\partial_{\alpha\beta} = (\gamma^m)_{\alpha\beta} \partial_m$ .

The  $N=5$  spinor derivatives are

$$D_{a\alpha} = \partial_{a\alpha} + i\theta_a^\beta \partial_{\alpha\beta}, \quad \partial_{a\alpha} \theta_b^\beta = \delta_{ab} \delta_\alpha^\beta. \quad (2.2)$$

The  $\text{SO}(5)/\text{U}(1) \times \text{U}(1)$  vector harmonics can be defined as components of the real orthogonal matrix

$$U_a^K = (U_a^{(1,1)}, U_a^{(1,-1)}, U_a^{(0,0)}, U_a^{(-1,1)}, U_a^{(-1,-1)}) \quad (2.3)$$

where  $a$  is the  $\text{SO}(5)$  vector index and  $K = 1, 2, \dots, 5$  corresponds to given combinations of the  $\text{U}(1) \times \text{U}(1)$  charges. These harmonics satisfy the following conditions:

$$U_a^K U_a^L = g^{KL} = g^{LK}, \quad g^{KL} U_a^K U_b^L = \delta_{ab}, \quad (2.4)$$

$$g^{15} = g^{24} = g^{33} = 1, \quad g^{11} = g^{12} = \dots = g^{45} = g^{55} = 0,$$

where  $g^{LK}$  is the antidiagonal symmetric constant metric in the space of charged indices.

In accordance with a general harmonic approach [13], we introduce the following harmonic derivatives:

$$\partial^{KL} = U_a^K g^{LM} \frac{\partial}{\partial U_a^M} - U_a^L g^{KM} \frac{\partial}{\partial U_a^M} = -\partial^{LK}, \quad (2.5)$$

$$[\partial^{IJ}, \partial^{KL}] = g^{JK} \partial^{IL} + g^{IL} \partial^{JK} - g^{IK} \partial^{JL} - g^{JL} \partial^{IK}, \quad (2.6)$$

which form generators of the  $\text{SO}(5)$  Lie algebra. For instance, the three charged harmonic derivatives

$$\begin{aligned} \partial^{12} &= \partial^{(2,0)} = U_a^{(1,1)} \partial / \partial U_a^{(-1,1)} - U_a^{(1,-1)} \partial / \partial U_a^{(-1,-1)}, \\ \partial^{13} &= \partial^{(1,1)} = U_a^{(1,1)} \partial / \partial U_a^{(0,0)} - U_a^{(0,0)} \partial / \partial U_a^{(-1,-1)}, \\ \partial^{23} &= \partial^{(1,-1)} = U_a^{(1,-1)} \partial / \partial U_a^{(0,0)} - U_a^{(0,0)} \partial / \partial U_a^{(-1,1)} \end{aligned} \quad (2.7)$$

satisfy the commutation relation

$$[\partial^{(1,-1)}, \partial^{(1,1)}] = \partial^{(2,0)}. \quad (2.8)$$

The Cartan charges of two  $\text{U}(1)$  groups are described by the neutral harmonic derivatives

$$\begin{aligned} \partial_1^0 &= \partial^{15} + \partial^{24}, & \partial_1^0 U_a^{(p,q)} &= p U_a^{(p,q)}, \\ \partial_2^0 &= \partial^{15} - \partial^{24}, & \partial_2^0 U_a^{(p,q)} &= q U_a^{(p,q)}. \end{aligned} \quad (2.9)$$

These charges arise in commutators of derivatives with opposite charges, for instance,

$$\begin{aligned} [\partial^{13}, \partial^{35}] &= [\partial^{(1,1)}, \partial^{(-1,-1)}] = \partial^{15} = \frac{1}{2}(\partial_1^0 + \partial_2^0), \\ [\partial^{23}, \partial^{34}] &= [\partial^{(1,-1)}, \partial^{(-1,1)}] = \partial^{24} = \frac{1}{2}(\partial_1^0 - \partial_2^0). \end{aligned} \quad (2.10)$$

Let us define the harmonic projections of the  $N=5$  Grassmann coordinates

$$\theta_\alpha^K = \theta_{a\alpha} U_a^K = (\theta_\alpha^{(1,1)}, \theta_\alpha^{(1,-1)}, \theta_\alpha^{(0,0)}, \theta_\alpha^{(-1,1)}, \theta_\alpha^{(-1,-1)}). \quad (2.11)$$

One can exclude two Grassmann coordinates  $\theta_\beta^{(-1,-1)}$  and  $\theta_\beta^{(-1,1)}$  and define the  $N = 5$  analytic superspace  $\zeta = (x_A^m, \theta_\alpha^{(1,1)}, \theta_\alpha^{(1,-1)}, \theta_\alpha^{(0,0)})$  with only three spinor coordinates and the shifted vector coordinate

$$x_A^m = x^m + i(\theta^{(1,1)}\gamma^m\theta^{(-1,-1)}) + i(\theta^{(1,-1)}\gamma^m\theta^{(-1,1)}). \quad (2.12)$$

The general superfields in the analytic coordinates depend also on additional spinor coordinates  $\theta_\alpha^{(-1,1)}$  and  $\theta_\alpha^{(-1,-1)}$ .

The harmonized partial spinor derivatives are

$$\begin{aligned} \partial_\alpha^{(-1,-1)} &= \partial/\partial\theta^{(1,1)\alpha}, & \partial_\alpha^{(-1,1)} &= \partial/\partial\theta^{(1,-1)\alpha}, & \partial_\alpha^{(0,0)} &= \partial/\partial\theta^{(0,0)\alpha}, \\ \partial_\alpha^{(1,1)} &= \partial/\partial\theta^{(-1,-1)\alpha}, & \partial_\alpha^{(1,-1)} &= \partial/\partial\theta^{(-1,1)\alpha}. \end{aligned} \quad (2.13)$$

An ordinary complex conjugation connects harmonics of the opposite  $U(1)$  charges

$$\overline{U_a^{(1,1)}} = U_a^{(-1,-1)}, \quad \overline{U_a^{(1,-1)}} = U_a^{(-1,1)}, \quad \overline{U_a^{(0,0)}} = U_a^{(0,0)}. \quad (2.14)$$

We use mainly the combined conjugation  $\sim$  in the harmonic superspace

$$\begin{aligned} \widetilde{U_B^{(p,q)}} &= U^{(p,-q)}, & \widetilde{\theta_\alpha^{(p,q)}} &= \theta_\alpha^{(p,-q)}, \\ (\theta_\alpha^{(p,q)}\theta_\beta^{(s,r)})^\sim &= \theta_\beta^{(s,-r)}\theta_\alpha^{(p,-q)}, & \widetilde{f(x_A)} &= \bar{f}(x_A), \end{aligned} \quad (2.15)$$

where  $\bar{f}$  is an ordinary complex conjugation.

One can define the combined conjugation for the harmonic derivatives, for instance,

$$(\partial^{(\pm 1,1)}A)^\sim = \partial^{(\pm 1,-1)}\tilde{A}, \quad (\partial^{(\pm 2,0)}A)^\sim = -\partial^{(\pm 2,0)}\tilde{A}. \quad (2.16)$$

The analytic integral measure contains partial derivatives on the analytic spinor coordinates (2.13)

$$\begin{aligned} d\mu^{(-4,0)} &= -\frac{1}{64}d^3x_A(\partial^{(-1,-1)})^2(\partial^{(-1,1)})^2(\partial^{(0,0)})^2 = d^3x_A d^6\theta^{(-4,0)}, \\ \int d^6\theta^{(-4,0)}(\theta^{(1,1)})^2(\theta^{(1,-1)})^2(\theta^{(0,0)})^2 &= 1. \end{aligned} \quad (2.17)$$

It is pure imaginary

$$(d\mu^{(-4,0)})^\sim = -d\mu^{(-4,0)}, \quad (d^6\theta^{(-4,0)})^\sim = -d^6\theta^{(-4,0)}. \quad (2.18)$$

The harmonic and spinor derivatives can be rewritten in the analytic coordinates

$$\begin{aligned} \mathcal{D}^{(1,1)} &= \partial^{(1,1)} - i(\theta^{(1,1)}\gamma^m\theta^{(0,0)})\partial_m - \theta^{(0,0)\alpha}\partial_\alpha^{(1,1)} + \theta^{(1,1)\alpha}\partial_\alpha^{(0,0)}, \\ \mathcal{D}^{(1,-1)} &= \partial^{(1,-1)} - i(\theta^{(1,-1)}\gamma^m\theta^{(0,0)})\partial_m - \theta^{(0,0)\alpha}\partial_\alpha^{(1,-1)} + \theta^{(1,-1)\alpha}\partial_\alpha^{(0,0)}, \end{aligned} \quad (2.19)$$

$$\begin{aligned} \mathcal{D}^{(2,0)} &= [\mathcal{D}^{(1,-1)}, \mathcal{D}^{(1,1)}] = \partial^{(2,0)} - 2i(\theta^{(1,1)}\gamma^m\theta^{(1,-1)})\partial_m - \theta^{(1,-1)\alpha}\partial_\alpha^{(1,1)} + \theta^{(1,1)\alpha}\partial_\alpha^{(1,-1)}, \\ D_\alpha^{(-1,-1)} &= \partial_\alpha^{(-1,-1)} + 2i\theta^{(-1,-1)\beta}\partial_{\alpha\beta}, & D_\alpha^{(-1,1)} &= \partial_\alpha^{(-1,1)} + 2i\theta^{(-1,1)\beta}\partial_{\alpha\beta}, \\ D_\alpha^{(0,0)} &= \partial_\alpha^{(0,0)} + i\theta^{(0,0)\beta}\partial_{\alpha\beta}, & D_\alpha^{(1,1)} &= \partial_\alpha^{(1,1)}, & D_\alpha^{(1,-1)} &= \partial_\alpha^{(1,-1)}. \end{aligned} \quad (2.20)$$

The analytic superfields  $\Lambda(\zeta, U)$  depend on harmonics and the analytic coordinates and satisfy the conditions

$$D_\alpha^{(1,\pm 1)}\Lambda = 0. \quad (2.21)$$

The harmonic derivatives  $\mathcal{D}^{(1,\pm 1)}$ ,  $\mathcal{D}^{(2,0)}$  commute with  $D_\alpha^{(1,\pm 1)}$  and preserve the Grassmann analyticity.

## 2.1 $N=5$ superconformal transformations

The superconformal  $D=3, N=5$  transformations can be defined by analogy with the corresponding  $D=4$  HSS superconformal transformations [13]. For instance, the special conformal K-transformations in the  $N=5$  analytic superspace are

$$\begin{aligned}\delta_k x_A^{\alpha\beta} &= k_{\gamma\rho} x_A^{\alpha\gamma} x_A^{\beta\rho}, & \delta_k \theta^{(0,0)\alpha} &= x_A^{\alpha\beta} \theta^{(0,0)\gamma} k_{\beta\gamma}, \\ \delta_k \theta^{(1,\pm 1)\alpha} &= x_A^{\alpha\beta} \theta^{(1,\pm 1)\gamma} k_{\beta\gamma} + \frac{i}{2} (\theta^{(0,0)})^2 \theta^{(1,\pm 1)\beta} k_{\beta}^{\alpha},\end{aligned}\quad (2.22)$$

where  $k_{\alpha\beta} = (\gamma^m)_{\alpha\beta} k_m$  are the corresponding parameters.

The K- and SO(5) transformations of harmonics have a similar form

$$\begin{aligned}(\delta_k + \delta_\omega) U_a^{(0,0)} &= -\lambda^{(1,1)} U_a^{(-1,-1)} - \lambda^{(1,-1)} U_a^{(-1,1)}, & (\delta_k + \delta_\omega) U_a^{(-1,\pm 1)} &= 0, \\ (\delta_k + \delta_\omega) U_a^{(1,\pm 1)} &= \lambda^{(1,\pm 1)} U_a^{(0,0)} + \lambda^{(2,0)} U_a^{(-1,\pm 1)},\end{aligned}\quad (2.23)$$

where

$$\lambda^{(1,\pm 1)} = 2ik_{\alpha\beta} \theta^{(1,\pm 1)\alpha} \theta^{(0,0)\beta} + U_a^{(1,\pm 1)} U_b^{(0,0)} \omega_{ab}, \quad \lambda^{(2,0)} = \mathcal{D}^{(1,-1)} \lambda^{(1,1)}, \quad (2.24)$$

and  $\omega_{ab}$  are the SO(5) parameters.

The S-supersymmetry transformation can be obtained via the Lie bracket of the special conformal and the Q-supersymmetry transformations  $\delta_\eta = [\delta_k, \delta_\epsilon]$ , where

$$\begin{aligned}\delta_\epsilon x_A^m &= -iU_a^{(0,0)} (\epsilon_a \gamma^m \theta^{(0,0)}) - 2iU_a^{(-1,1)} (\epsilon_a \gamma^m \theta^{(1,-1)}) \\ &\quad - 2iU_a^{(-1,-1)} (\epsilon_a \gamma^m \theta^{(1,1)}), & \delta_\epsilon \theta^{(p,q)\alpha} &= U_a^{(p,q)} \epsilon_a^\alpha.\end{aligned}\quad (2.25)$$

It is easy to see that the analytic-superspace integral measure  $d^3 x_A d^6 \theta^{(-4,0)} dU$  is invariant with respect to these superconformal transformations.

## 3 Chern-Simons-type model in $N=5$ analytic superspace

Using a formal analogy with the  $D=4, N=3$  HSS [13] we introduce the  $D=3, N=5$  analytic matrix gauge prepotentials for our harmonic derivatives

$$V^{(1,1)}(\zeta, U), \quad V^{(1,-1)}(\zeta, U), \quad V^{(2,0)}(\zeta, U). \quad (3.1)$$

The reality conditions for these prepotentials are

$$(V^{(1,1)})^\dagger = -V^{(1,-1)}, \quad (V^{(2,0)})^\dagger = V^{(2,0)}, \quad (3.2)$$

where the Hermitian conjugation  $\dagger$  includes the  $\sim$ -conjugation of the matrix elements. The infinitesimal gauge transformations of these gauge superfields depend on the analytic anti-Hermitian gauge parameter  $\Lambda$

$$\delta_\Lambda V^{(1,\pm 1)} = D^{(1,\pm 1)} \Lambda + [V^{(1,\pm 1)}, \Lambda], \quad \delta_\Lambda V^{(2,0)} = D^{(2,0)} \Lambda + [V^{(2,0)}, \Lambda]. \quad (3.3)$$

The covariant harmonic derivatives preserving the G-analyticity have the following form:

$$\nabla^{(1,\pm 1)} = \mathcal{D}^{(1,1)} + V^{(1,\pm 1)}, \quad \nabla^{(2,0)} = \mathcal{D}^{(2,0)} + V^{(2,0)}, \quad (3.4)$$

The analytic CS-type action can be constructed in terms of three gauge prepotentials

$$\begin{aligned}
S = & \frac{i}{g^2} \int dU d^3 x_A d^6 \theta^{(-4,0)} \text{Tr} \{ V^{2,0} (\mathcal{D}^{(1,1)} V^{(1,-1)} - \mathcal{D}^{(1,-1)} V^{(1,1)}) \\
& + V^{1,1} (\mathcal{D}^{(1,-1)} V^{(2,0)} - \mathcal{D}^{(2,0)} V^{(1,-1)}) + V^{1,-1} (\mathcal{D}^{(1,1)} V^{(2,0)} - \mathcal{D}^{(2,0)} V^{(1,1)}) \\
& - 2V^{2,0} [V^{(1,-1)}, V^{(1,1)}] + V^{(2,0)} V^{(2,0)} \},
\end{aligned} \tag{3.5}$$

where  $g$  is the dimensionless coupling constant.

The corresponding superfield gauge equations of motion are

$$F^{3,1} = \mathcal{D}^{(1,1)} V^{(2,0)} - \mathcal{D}^{(2,0)} V^{(1,1)} + [V^{(1,1)}, V^{(2,0)}] = 0, \tag{3.6}$$

$$V^{(2,0)} = \mathcal{D}^{(1,-1)} V^{(1,1)} - \mathcal{D}^{(1,1)} V^{(1,-1)} + [V^{(1,-1)}, V^{(1,1)}] \equiv \hat{V}^{(2,0)}. \tag{3.7}$$

The last superfield can be composed algebraically in terms of two other prepotentials. Using the substitution  $V^{(2,0)} \rightarrow \hat{V}^{(2,0)}$  in (3.5) we can obtain the alternative form of the action with only two independent prepotentials  $V^{1,1}$  and  $V^{1,-1}$

$$\begin{aligned}
S_2 = & \frac{i}{g^2} \int dU d\mu^{(-4,0)} \text{Tr} \{ V^{1,-1} \mathcal{D}^{(2,0)} V^{(1,1)} - V^{1,1} \mathcal{D}^{(2,0)} V^{(1,-1)} \\
& - (\mathcal{D}^{(1,-1)} V^{(1,1)} - \mathcal{D}^{(1,1)} V^{(1,-1)} + [V^{(1,-1)}, V^{(1,1)}])^2 \}.
\end{aligned} \tag{3.8}$$

It is evident that the superfield action (3.5) is invariant with respect to the sixth supersymmetry transformation defined on our gauge potentials

$$\delta_6 (V^{(1,\pm 1)}, V^{(2,0)}) = \epsilon_6^\alpha D_\alpha^{(0,0)} (V^{(1,\pm 1)}, V^{(2,0)}), \tag{3.9}$$

where  $\epsilon_6^\alpha$  are the corresponding spinor parameters. Thus, our superfield CST gauge model possesses the  $D=3, N=6$  supersymmetry.

The CST actions (3.5) and (3.8) are formally similar to the HSS actions of the  $\text{SYM}_4^3$  theory[13] (or to the action of the dimensionally reduced  $\text{SYM}_3^6$  theory), although the Grassmann dimensions of analytic superspaces in these two types of models are different. In the next section, we analyze the field-component structure of our superfield model.

## 4 Harmonic component fields in the $N=6$ Chern-Simons-type model

The abelian form of the action  $S_2$  (3.8) contains  $\sim$ -conjugated abelian prepotentials

$$\begin{aligned}
S_2^0 = & i \int dU d\mu^{(-4,0)} \{ V^{1,-1} \mathcal{D}^{(2,0)} V^{(1,1)} - V^{1,1} \mathcal{D}^{(2,0)} V^{(1,-1)} \\
& - (\mathcal{D}^{(1,-1)} V^{(1,1)} - \mathcal{D}^{(1,1)} V^{(1,-1)})^2 \}.
\end{aligned} \tag{4.1}$$

The Grassmann and harmonic decompositions of the imaginary gauge superfield parameter have the following form:

$$\begin{aligned}
\Lambda(\zeta, U) = & i[a + U_b^{(0,0)} a^b] + \theta^{(1,1)\alpha} U_b^{(-1,-1)} \rho_\alpha^b + \theta^{(1,-1)\alpha} U_b^{(-1,1)} \bar{\rho}_\alpha^b + \theta^{(0,0)\alpha} [\beta_\alpha + U_b^{(0,0)} \beta_\alpha^b] \\
& + (\theta^{(0,0)})^2 [d + U_a^{(0,0)} d_a] + (\theta^{(1,1)} \theta^{(0,0)}) U_b^{(-1,-1)} f^b + (\theta^{(1,-1)} \theta^{(0,0)}) U_b^{(-1,1)} \bar{f}^b \\
& + (\theta^{(1,1)\alpha} \theta^{(0,0)\beta}) U_b^{(-1,-1)} g_{\alpha\beta}^b + (\theta^{(1,-1)\alpha} \theta^{(0,0)\beta}) U_b^{(-1,1)} \bar{g}_{\alpha\beta}^b \\
& + (\theta^{(0,0)})^2 \theta^{(1,1)\alpha} U_b^{(-1,-1)} \pi_\alpha^b - (\theta^{(0,0)})^2 \theta^{(1,-1)\alpha} U_b^{(-1,1)} \bar{\pi}_\alpha^b + O(U^2),
\end{aligned} \tag{4.2}$$

where all coefficients depend on  $x_A^m$ . Bilinear in harmonics terms and the corresponding  $\theta$  terms are omitted, and the condition  $\Lambda^\sim = -\Lambda$  is used in this formula.

The pure gauge degrees of freedom in the prepotential  $V^{(1,1)}$  can be eliminated by the transformation  $\delta V^{(1,1)} = \mathcal{D}^{(1,1)}\Lambda$ . In the WZ-gauge, the superfield parameter is reduced to  $\Lambda_{WZ} = ia(x_A)$ , but the corresponding gauge superfield still has an infinite number of component fields

$$\begin{aligned} V_{WZ}^{(1,1)} = & U_a^{(1,1)}v^a + i\theta^{(1,1)\alpha}\mu_\alpha + i\theta^{(0,0)\alpha}U_a^{(1,1)}\nu_\alpha^a - (\theta^{(1,1)}\gamma^m\theta^{(0,0)})(A_m + iB_m) \\ & + (\theta^{(1,1)}\gamma^m\theta^{(1,-1)})U_a^{(-1,1)}C_m^a + i(\theta^{(1,1)}\theta^{(0,0)})(S + U_a^{(0,0)}S^a) + (\theta^{(0,0)})^2U_a^{(1,1)}b^a \\ & + (\theta^{(1,1)})^2U_a^{(-1,-1)}h^a + \theta^{(1,1)\alpha}\theta^{(1,-1)\beta}\theta^{(0,0)\gamma}U_a^{(-1,1)}\Psi_{(\alpha\beta\gamma)}^a + (\theta^{(1,-1)}\theta^{(0,0)})\theta^{(1,1)\alpha}U_a^{(-1,1)}\xi_\alpha^a \\ & + (\theta^{(1,1)}\theta^{(0,0)})\theta^{(1,-1)\alpha}U_a^{(-1,1)}\lambda_\alpha^a + (\theta^{(0,0)})^2\theta^{(1,1)\alpha}[\psi_\alpha + U_a^{(0,0)}\psi_\alpha^a] \\ & + (\theta^{(1,1)})^2(\theta^{(0,0)})^2U_a^{(-1,-1)}F^a + (\theta^{(1,1)}\theta^{(1,-1)})(\theta^{(0,0)})^2U_a^{(-1,1)}G^a \\ & + (\theta^{(1,1)}\gamma^m\theta^{(1,-1)})(\theta^{(0,0)})^2U_a^{(-1,1)}G_m^a + O(U^2), \end{aligned} \quad (4.3)$$

where again bilinear in harmonics terms and the corresponding Grassmann terms are omitted for brevity. The basic fields of this supermultiplet

$$v^a, \mu_\alpha, \nu_\alpha^a, A_m, B_m, S, S^a \quad (4.4)$$

are real and other fields are complex. Note that  $A_m$  is the gauge field ( $\delta A_m = \partial_m a$ ), but we have no gauge parameter for the second vector field  $B_m$ . Dimensions of these off-shell fields are

$$\begin{aligned} [v^a] = 0, \quad [\mu_\alpha] = [\nu_\alpha^a] = \frac{1}{2}, \quad [A_m] = [B_m] = [S] = [S^a] = [b^a] = [C_m^a] = 1, \\ [\psi_\alpha] = [\psi_\alpha^a] = [\xi_\alpha^a] = [\lambda_\alpha^a] = [\Psi_{\alpha\beta\gamma}^a] = \frac{3}{2}, \quad [F^a] = [G^a] = [G_m^a] = 2. \end{aligned} \quad (4.5)$$

In the representation (4.3), we omit an infinite number of auxiliary bosonic and fermionic fields  $F^{a_1 \dots a_k}$  ( $k \geq 2$ ) with multiple vector SO(5) indices.

The component Lagrangian for the SO(5) invariant fields has the following form:

$$\begin{aligned} L_0 = & 3\varepsilon^{mnr}A_m\partial_nA_r + \varepsilon^{mnr}B_m\partial_nB_r - 4S\partial^mB_m - i\psi^\alpha\psi_\alpha - i\bar{\psi}^a\bar{\psi}_a + i\psi^\alpha\bar{\psi}_\alpha \\ & + \frac{3i}{2}(\psi^\alpha + \bar{\psi}^\alpha)\partial_{\alpha\beta}\mu^\beta - \frac{i}{4}\partial_\beta^\alpha\mu^\beta\partial_{\alpha\gamma}\mu^\gamma. \end{aligned} \quad (4.6)$$

The corresponding equations of motion have the pure gauge solution for  $A_m$  and the nontrivial solution for the vector field  $B_m$ . The scalar field  $S$  and the noncanonical fermionic field  $\mu^\alpha$  satisfy the free equations  $\partial^m\partial_mS = 0$ ,  $\partial^m\partial_m\mu^\alpha = 0$ . The on-shell construction for the auxiliary fermionic field is  $\psi_\alpha = \frac{3}{2}\partial_{\alpha\beta}\mu^\beta$ .

It is not difficult to construct the Lagrangian for the SO(5) vector fields from (4.3) using the superfield action (4.1). The basic SO(5) vector fields satisfy the free equations  $\partial^m\partial_mv^a = \partial^m\partial_mS^a = \partial^m\partial_m\nu_\alpha^a = 0$ , and the on-shell complex auxiliary fields can be composed from these basic real fields

$$\begin{aligned} b^a = -\frac{3i}{2}S^a, \quad h^a = 2iS^a, \quad C_m^a = 3i\partial_mv^a, \quad F^a = -\frac{1}{2}\partial^m\partial_mv^a, \quad G^a = -\frac{1}{3}\partial^m\partial_mv^a, \\ G_m^a = \partial_mS^a, \quad \xi_\alpha^a = \partial_{\alpha\beta}\nu^{a\beta}, \quad \lambda_\alpha^a = -\frac{5}{3}\partial_{\alpha\beta}\nu^{a\beta}, \quad \psi_\alpha^a = \frac{4}{3}\partial_{\alpha\beta}\nu^{a\beta}, \quad \Psi_{\alpha\beta\gamma}^a = -\partial_{(\alpha\beta}\nu_{\gamma)}^a. \end{aligned}$$

All abelian auxiliary fields with more than two SO(5) indices vanish on-shell.

Thus, the solutions for the superfield N=5 gauge prepotentials contain the nontrivial vector, scalar and fermion fields in distinction with the pure gauge superfield solutions of the N=1, 2 and 3 supersymmetric Chern-Simons theories.

We plan to analyze the nonabelian interactions of the Chern-Simons gauge field  $A_m$  with the fields  $v^a, \mu_\alpha, \nu_\alpha^a, B_m, S$  and  $S^a$  (all fields in the adjoint representation of the gauge group) using the algebraic auxiliary field equations in our  $N=6$  Chern-Simons-type model. The superfield representation may be useful for quantum calculations.

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